

# Tools For Determining Optimal Pricing of Pari-mutuel Wagers

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# Economics and Race-Track Betting

Economists have studied race-track wagering since at least 1950s, in multiple ways

- **theoretically**

- mathematical “models” of bettor behavior

- **empirically**

- statistical (econometric) techniques
- calibration and simulation techniques
- laboratory experimental techniques
- field experimental techniques

# Economics and Race-Track Betting

Economic research: the goal of theory and empirical tests of theory, is to understand underlying forces for economic behavior

- (how) does “treatment A affect B”

Goal of research isn't to determine revenue maximizing takeout, but methods and results are pertinent to that task

- how could the methods/frameworks (and results) be utilized to assist horse racing in determining optimal takeout?

# Economics and Race-Track Betting

Pari-mutuel odds aggregate diverse beliefs, or “subjective probabilities” of bettors that are based on diverse information

- just as we view asset market (stock) prices as doing

Asset market “efficiency”: all available information reflected in prices

- asset prices instantaneously reveal information of informed investors as a result of their trades
- subjective evaluations reflected in prices should on average equal objective values

=> No (systematic) unexploited profit opportunities

# Economics and Race-Track Betting

## Why are economists interested in race-track betting markets?

- A. Great environment for testing efficiency/rationality of asset markets – all information reflected in market prices: pari-mutuel odds
  - i) race-track bets have well-defined termination points at which value of bet/outcome certain, known (unlike stock markets)
  - ii) repeated, allows for rapid learning from mistakes, and tendency towards efficiency/rationality
  - iii) superior testing environment vs. stock markets

Easy to test efficiency empirically – large sample of ex-post outcomes.

=> anomalies found, relative to implication of efficient markets

# Economics and Race-Track Betting

e.g. Thaler and Ziemba (1988, *Journal of Economic Perspectives*) summarize this literature

- i) studies find racetrack betting markets surprisingly efficient
- ii) market odds remarkably good estimates of ex-post winning frequencies
- iii) one quite robust anomaly: favorite/long-shot bias
- iv) evidence of Gambler's fallacy in various contexts

# Economics and Race-Track Betting

B. Develop theoretical explanations for observed inefficiencies, many of which generalize to other asset market environments - test them

- i) alternative bettor attitudes to risk
- ii) models of differentiated bettor types by skill, information set; strategic/non-competitive betting behavior by informed bettors
- iii) role of transactions costs - takeout, information acquisition costs
- iv) mechanism design – could modifications or extensions of the of pari-mutuel design itself ameliorate inefficiencies?

Limited access to proprietary data => lab experiments for testing.

# Economics and Race-Track Betting

Long tradition of this line of research, exploring why pari-mutuel subjective probabilities are biased forecasts of outcomes.

- takeout rate is typically a “parameter” - taken as given by the researcher,
- e.g. how takeout affects informed bettor decisions to enter pools, how much to bet into them (Terrell and Farmer (1996))
  - reduces entry into pools, and amount bet.

Limited access to data => experimental evidence



# Experimental Evidence

Laboratory experiments, use university students – proxies for “youth demand” in racing – controlled environments

- have not been designed to answer business policy questions regarding optimal takeout
- focused on information aggregation/efficiency and pari-mutuel mechanism design

# Experimental Evidence

Plott et al. (2002, Economic Theory)

- construct an abstract pari-mutuel game, provide participants with diverse information eg. true outcomes vs. crude info
- seek evidence for how efficiently market odds reflect “truth”
- observe bluffing - reduces efficiency - and waiting/learning - improves efficiency – also observe biases we see in field
- allowing secondary or re-sale markets in “tickets”  
improves informational efficiency, increased gross and net ticket sales

# Field Experiments

Subject field subjects to experimentation in lab, or conduct experiments in field, etc.

- Example in pari-mutuel research is Camerer (1998, *Journal of Political Economy*)
  - repeatedly placed (and canceled) bets of significant amounts of money to attempt to manipulate market odds at local track
  - do (temporary) bets affect prices appreciably?
  - no

# Field Experiments

In practice, these widely used to assist in revenue maximization policies in other industries

- economists hired to conduct field experiments in early days of advertising via search engines to determine revenue maximizing auction mechanism for ad pricing
- increased revenue by 27% in three months
- also in establishing revenue maximizing strategies for soliciting charitable contributions via “reward” structure

Feasible to experiment with entire menu of takeout rates, taking as given all other features of the environment, provided appropriate “controls” in place eg. repeated sampling, “blind” experiments.

# Demand for Wagering

Empirically, economists have tried to isolate the impact of takeout for wagering demand

- econometric studies

  - estimate the *marginal* impact of takeout, controlling for other factors relevant for handle

# Econometric Studies

Historically, each track a (local) monopoly, selling only conventional wagers

- simple econometric models sufficient to estimate total demand for conventional wagers as function of takeout
- estimate response of handle to takeout , controlling for real income per capita as a proxy for “budget”

$$\text{Gross Handle} = a * \text{takeout} + b * \text{per capita income} + \text{error}$$

# Econometric Studies

Recent studies have much more complex set of control or explanatory variables and decompose handle by wager type

- account for availability of alternatives (slots, lotteries), prices of alternatives, simulcasting card, etc.
- no longer in monopoly framework so “cross-price” elasticities important to account for
- are wagers substitutes/complements for each other, simo/wagers on other tracks, alternative gaming opportunities

# Econometric Studies

Estimates of the response of handle to takeout controlling for response to other variables, normalized as “price elasticities”

$$\text{Elasticity} = \% \Delta \text{ handle} / \% \Delta \text{ takeout}$$

“Unit free”, compare across goods, markets, time...



# Econometric Studies

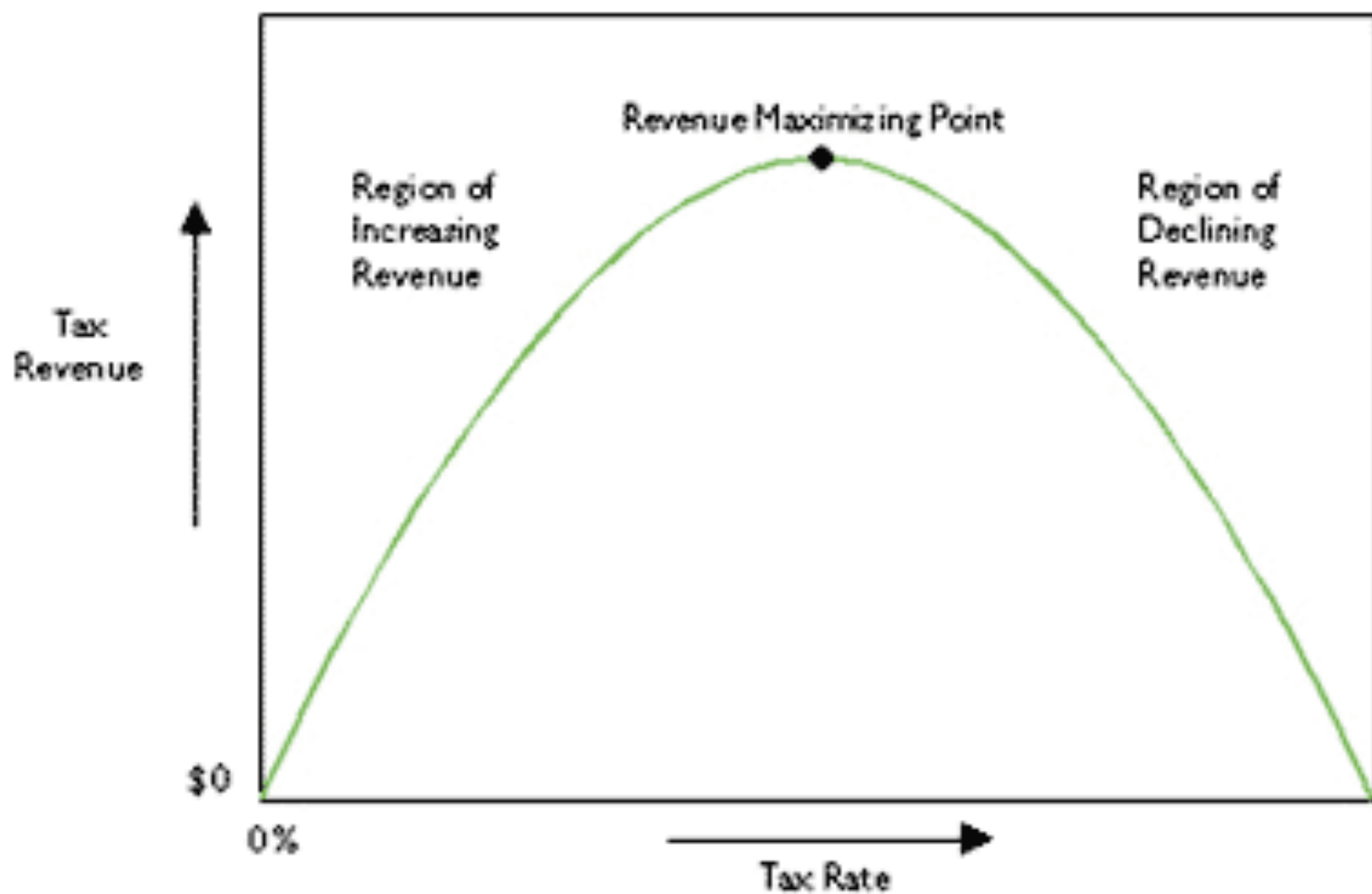
Elasticity of demand estimates directly relevant to revenue maximization problem

- since takeout rate is a fixed % of total income a “Laffer Curve” framework for analyzing revenue appropriate

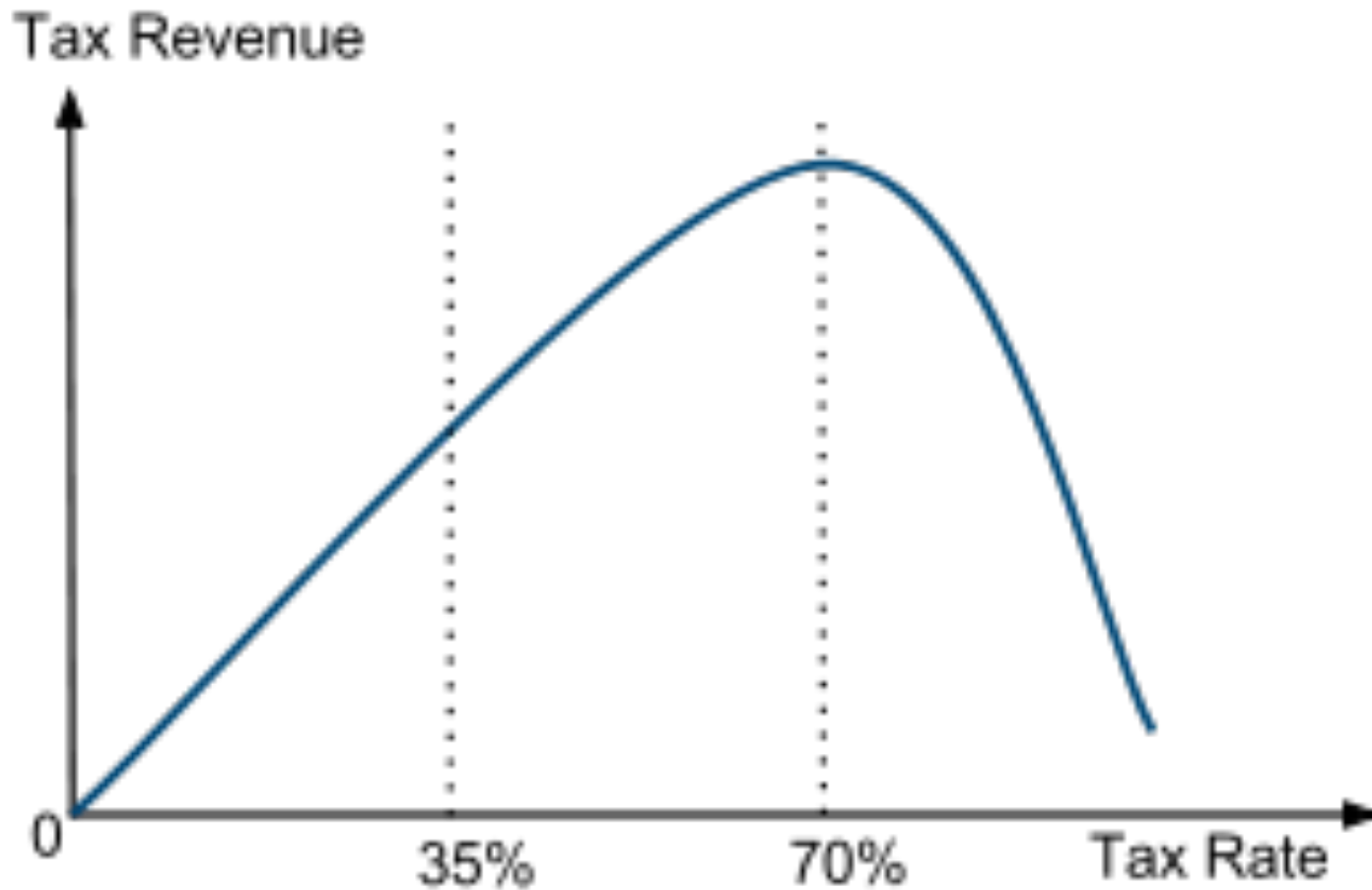
If tax rate is 0%, tax revenue is zero; if tax rate is 100%, tax base and revenue is zero

- somewhere between 0% and 100% is revenue maximizing
- where depends on the price elasticity of demand

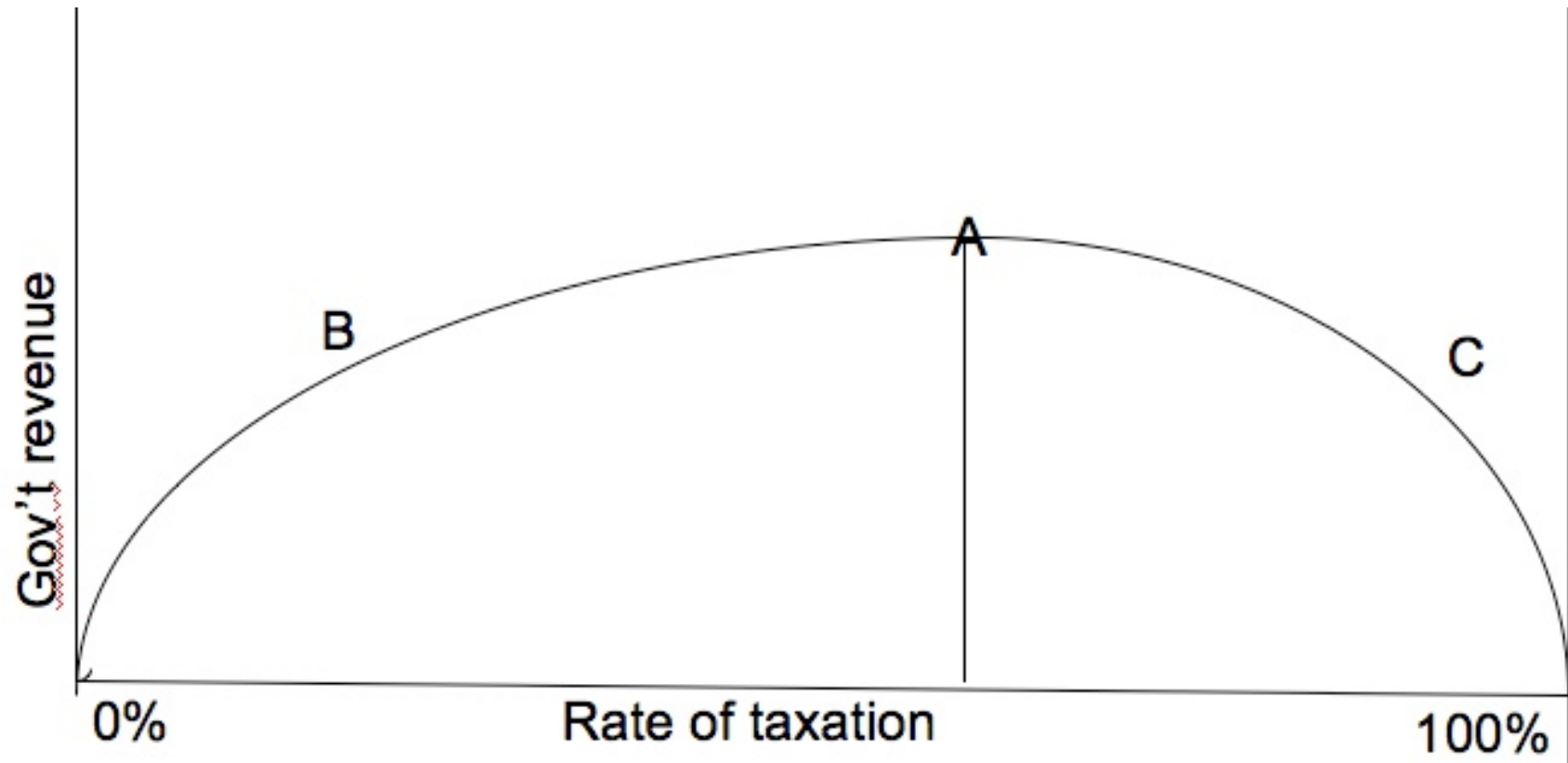
## The Laffer Curve



# The Laffer Curve II (labor income)



# Laffer Curve III



# Laffer Curve Shape

$$\text{Tax Revenue} = \text{Tax Rate} \times \text{Tax Base}$$

$$\% \Delta \text{ Tax Revenue} = \% \Delta \text{ Tax Rate} + \% \Delta \text{ Tax Base}$$

$$\% \Delta \text{ Tax Revenue} / \% \Delta \text{ Tax Rate} = 1 + \text{elasticity}$$

- At low rates, decline in tax base small for a given increase in tax rate, revenue increases (elasticity  $\geq -1$ )
- At inflection point, decline in tax base exactly offsets increase in tax rate, revenue unchanged (elasticity = -1)
- Thereafter, possibly due to “tax evasion”, decline in tax base outweighs increase in tax rate, revenue declines (elasticity  $< -1$ )

# Laffer Curve Shape

In pari-mutuel context, the %  $\Delta$  in handle arising from a given %  $\Delta$  in takeout is the elasticity of demand for wagering

- Econometric estimates of demand elasticity tell you “proximate location/which side” of the Laffer Curve you are on.

Almost uniformly the case that we’re on the “wrong side” – demand elasticities exceed -1.

# Econometric Studies

## Shortcomings

- elasticity estimates tell you where you are relative to the inflection point but not necessarily where the inflection point is
- aggregate data is all that's available typically, cannot discriminate between responses of bettor types
- institutional change limits sample periods that can be used though don't constrain use of the technique *per se*
- don't observe some prices, alternatives - rebates, off-shore accounts

Field Experiments could un-cover the entire Laffer Curve.

# Econometric Study Examples

Gruen (1976, Journal of Political Economy)

- 1940-1969 Aqueduct, annual data
- avoids introduction of off-track betting and exotic wagers
- controls for wartime years (pessimism), low unemployment (optimism), real per capita income (budget)

Finds strong positive income effect, strong negative price elasticity, implying takeout rates higher than revenue maximizing in sample

- by assuming specific form for market demand curve, calibrates model to show 1969 takeout rate 2.88 % points higher than “optimal” (17.16 vs. 14.88)



# Econometric Study Examples

## Thalheimer and Ali (1995, Management Science)

- 1960-1987, Turfway Park, River Downs and Lebanon Raceway, annual handle and annual attendance equations simultaneously estimated
- real per capita income, racing quality - purse size, own takeout rate is weighted average across wager types, substitute takeout rate for lottery, substitute product professional sporting dates, # racing dates
- handle is “own price” elastic at all three tracks varying from -2.85 to -3.09
- attendance is also “own price” elastic, though less so => wagering per on-track patron elastic re. takeout.

# Econometric Study Examples

Gramm et al (2007, American Journal of Economics and Sociology)

- pool cross-sectional data from twelve tracks, October – December 2002
- 2957 RACES analyzed
- five equations: Win, Place, Show, Exacta, Trifecta handle by race
- 32 independent variables per equation:
  - race quality/type variables, purse, takeout by wager type, availability of DD, P6 etc., competitiveness index, # betting interests, surface, carryover incidence, off-track, distance, number of simulcast races available within an hour...

All wagers and total handle are price elastic (except trifectas) varying from -2.4 to -3.6 and price elasticities DIFFER by wager including Win, Place, Show

# Calibration/Simulation

Could use the price elasticity estimates to calibrate a behavioral model and infer optimal price by wager

- historically elasticity estimates used to “calibrate” parameters of a very simple monopoly markup pricing model
- today we’d need a much richer model of industrial organization
  - oligopoly (industry with 100 firms or less) involving strategic pricing reactions
  - multiple products/wagers – complements and substitutes

Compare simulated data to actual data, identify weaknesses of model, modify, re-simulate etc. until prediction warrants use as a forecasting tool for alternative takeout rates.

# Or...

Use econometric model itself to forecast handle for alternative values of takeout

- one issue is how stable parameter estimates would be at alternative takeout rates very different from those in sample.

Subject to the “Lucas Critique” – when you change policy, underlying consumer behavior changes